TOPICS ON BOUNDARY-ELEMENT SOLUTIONS OF WAVE RADIATION-DIFFRACTION PROBLEMS

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Abstract

Two topics on the numerical solution of boundary-integral equations arising in linear wave-body interactions are discussed. The properties of a spectral technique for the solution of the integral equation are analyzed and compared to the conventional collocation method. It is shown that, using this technique, hydrodynamic forces predicted by the source-distribution method are identical to those obtained from the direct solution for the velocity potential. The second part of the paper investigates the numerical properties of a method which removes the effects of the irregular frequencies for bodies of general shape at a small computational and algorithmic overhead. Its performance is illustrated in the evaluation of the heave and sway hydrodynamic coefficients of a circle and a rectangle.

1. Introduction

The solution of boundary-integral equations for the evaluation of the linear wave loads on marine structures is a common task in today's practice. Its success is due to its algorithmic simplicity, the ease of describing the surface of a three-dimensional body by a collection of facets and the moderate size of the linear systems to be solved. These are illustrated by its widespread use by aerodynamicists [Hess and Smith (1966)].

In the presence of a free surface, the efficiency of the method relies on the fast evaluation of the wave-source potential which is a substantially more complex function to compute than its counterparts in an infinite fluid and an acoustic medium. Existing methods for the computation of its values and derivatives are hard to evaluate, since it is the performance of the integrated radiation-diffraction computer programs that is usually reported. For the three-dimensional computations reported in the present paper, a set of very efficient algorithms developed by Newman (1985a) for water of finite and infinite depth and coded in the subroutine FINGREEN have been utilized.

A distinct feature of wave boundary-integral equations are the " irregular " frequencies. They coincide with the eigenfrequencies of the interior Dirichlet or Neumann problems, and are known to introduce large errors in the predicted hydrodynamic forces, often over a substantial band of frequencies. A comprehensive analysis of the mathematical properties of boundary-integral equations, (with emphasis in acoustics), along with a survey of techniques used for the removal of the irregular frequencies, is given in the recent book of Colton and Kress (1983). The numerical aspects of boundary-integral, as well as finite-element, hybrid-integral and finite-element/boundary-integral methods in free-surface flows are reviewed by Mei (1978), Yeung (1982) and Euvrard (1983).

The first part of the paper analyzes the properties of a technique for solving boundary-integral equations. It is often quoted in the literature as the Galerkin method. In most implementations of the boundary-integral formulation, the body surface is approximated by a collection of N plane quadrilaterals or triangles. The conventional collocation method enforces the validity of the equation at a single point on each panel, usually the centroid. The method proposed here, averages the equation over each panel and avoids the need to select a collocation point. It belongs in the general category of "spectral" techniques which express the solution in terms of N basis functions, and then pre-integrate the product of the equation to be solved with each function of this set. Here, the i-th basis function has the value one on the i-th panel and zero on the rest of them.

The Galerkin technique has a set of interesting symmetry properties. The diagonal elements in the added-mass and damping matrices obtained from the source-distribution and the Green method are identical. The off-diagonal coefficients A_{ij}, B_{ij} obtained from one method are identical to the A_{ji}, B_{ji} coefficients which follow from the other. Moreover, the exciting forces obtained from the solution of the Green integral equation for the diffraction velocity potential, are identical to those obtained from the Haskind relations with the radiation velocity potential supplied by the source-distribution method, and vice versa. Analogous results do not hold in the collocation method.

Computations of the hydrodynamic coefficients for a semi-submerged spheroid and a truncated vertical circular cylinder have been carried out by Breit, Newman and Sclavounos (1985). No substantial difference from the predictions of the collocation method has been observed. Near the irregular frequencies, the Galerkin method led to a reduction of the error and the frequency bandwidth over which it occurs. In principle, it requires an additional numerical integration for the evaluation of the infuence of panel i on panel j. In Section 2, an algorithm is proposed which reduces substantially this overhead, while maintaining the main features of the Galerkin technique.

For the Green integral equation, the irregular frequencies coincide with the eigenfrequencies of the interior Dirichlet problem. Both in two and three dimensions, they can be suppressed by adding an artificial lid on the interior free surface as suggested by Ohmatsu (1975) in connection with the source-distribution method, and by Kleinman (1982) for the Green integral equation. This approach is effective, but may lead to a substantial increase in the computational effort, especially in three dimensions. Ogilvie and Shin (1977) suggested an alternative route by adding a wave source at the origin of the coordinate system, assumed to lie on the interior free surface, acting as an absorber of the energy associated with the interior Dirichlet eigensolutions. This approach was implemented in two-dimensions and was shown to successfully remove the first irregular frequency, at a small additional computational effort. Ursell (1982) later established that any number of irregular frequencies can be removed if a sufficient number of singularities are added at the origin. No implementation of this method in three dimensions is known to the authors.

Related work in acoustics predates the studies of marine hydrodynamicists by about a decade. Refe rences to early studies can be found in Colton and Kress (1983). Burton and Miller (1971) proposed a methodology which exploits the different location of the irregular frequencies of integral equations of the first and second kind. The linear combination of two such equations for the exterior Neumann problem has a unique solution on the entire real frequency axis, and thus is free of irregular frequencies. The condition for this to occur is that one of the two equations must be multiplied by the imaginary unit times a positive constant. It turns out that the associated interior homogeneous problem is of mixed Dirichlet and Neumann type, the two being 90 degrees out of phase. Non-trivial eigen solutions cannot exist since on the interior boundary the normal velocity is 90 degrees out of phase from the velocity potential, or in phase with the pressure. Thus energy may flow out of the interior domain preventing the persistence of eigensolutions. The direction of the energy flux is controlled by the sign of the constant used in the linear combination of the two equations.

Euvrard, Jami, Lenoir and Martin (1981) were the first to adapt this methodology to wave-body interaction problems. Their formulation combines a layer of finite elements in the fluid domain which encloses the body boundary, with a boundary-integral representation in the exterior domain analogous to that outlined in the preceding paragraph. Computations of the hydrodynamic coefficients of three-dimensional bodies were found free of the effects of irregular frequencies.

In the present paper the method of Burton and Miller (1971) is applied to the solution of the radiation problem. It corresponds to the limit of a *finite-element* layer of zero thickness in the scheme of Euvrard et al. A circle and a rectangle have been analyzed. Their boundary has been approximated by straight segments and the equation has been satisfied at their mid-point, according to the collocation method. In both the heave and sway added mass and damping coefficients, errors have been suppressed at and in the vicinity of the irregular frequencies.

The associated computational and algorithmic overhead is small, since the effort involved in the computation of the second derivatives of the wave source potential (they appear in the kernel of the equation of the first kind) is not large relative to that required by its value and first derivatives. In deep water this overhead is negligible because of the existence of recurrence relations which relate derivatives of high to those of lower order. Moreover, the size of the linear system is unaffected by the superposition of two equations over the same number of panels.

The method is currently being extended to three-dimensional problems where the irregular-frequency effects are generally less pronounced, and is expected to be effective for bodies of general shape. This is believed to be the case because the addition of the integral equation of the first kind to the Green equation essentially corresponds to an additional singularity distribution on the actual body surface rather than at a selected set of points internal to its boundary.

2. The Galerkin method

We are interested to evaluate the hydrodynamic pressure on the surface of a body interacting with regular free-surface waves. Linearity, and the assumption of irrotational flow, allows the reduction of the problem to the evaluation of a velocity potential $\phi(x)$ which satisfies the Laplace equation in the fluid domain, the linear free-surface condition, a prescribed normal gradient $v(x)$ on the body surface, the vanishing of its gradient at large depths and a radiation condition at infinity.

Two boundary-integral formulations are common, and both can be derived from Green's theorem. According to the source-distribution method, $\phi(x)$ is represented by a distribution of wave sources on the body surface,

$$\phi(x) = \int_S \sigma(\xi) G(x;\xi) d\xi \qquad (2.1)$$

where $G(x;\xi)$ is the wave-source potential at the point x due a unit source located at the point ξ. Enforcing the normal velocity on the body boundary leads to an integral equation for the strength $\sigma(x)$ of the source distribution,

$$-\frac{1}{2}\sigma(x) + \int_S \sigma(\xi)\frac{\partial G(x;\xi)}{\partial n_x} d\xi = \frac{\partial \phi}{\partial n_x} = V(x) \qquad (2.2)$$

The application of a different variant of Green's theorem leads to an integral equation with the velocity potential itself as the unknown,

$$-\frac{1}{2}\phi(x) + \int_S \phi(\xi)\frac{\partial G(\xi;x)}{\partial n_\xi} d\xi = \int_S \frac{\partial \phi}{\partial n_\xi} G(\xi;x) d\xi \qquad (2.3)$$

Equations (2.2) and (2.3) are adjoint Fredholm integral equations of the second kind, since the kernel of (2.2) is obtained from the transposition of the arguments of the kernel of (2.3). The preference of the one versus the other depends on the application for which they are being considered. If, for example, the flow velocities are required then equations (2.1)-(2.2) appear to be at an advantage since the evaluation of the second derivatives of G is not necessary. If, on the other hand, only quantities dependent on the velocity potential are needed, then (2.3) must be preferred due to the reduced storage requirements.

Their numerical solution is usually obtained by approximating the body boundary by a collection of plane quadrilaterals or triangles, assuming that the unknown functions have constant values on each panel. The equations are enforced at a set of collocation points usually taken to be the centroids. The Galerkin technique, proposed here, averages instead the equations over each panel. In discrete form, equations (2.1)-(2.2) become,

$$\phi_i = \frac{1}{A_i}\sum_{j=1}^{N} \sigma_j \int_{S_i} dx \int_{S_j} d\xi\, G(x;\xi)$$

$$= \frac{1}{A_i}\sum_{j=1}^{N} G_{ij}\sigma_j \qquad (2.4)$$

$$G_{ij} = \int_{S_i} dx \int_{S_j} d\xi\, G(\xi;x) \qquad (2.5)$$

$$-\frac{1}{2}\sigma_i A_i + \sum_{j=1}^{N} D^{(S)}_{ij}\sigma_j = V_i A_i, \quad i = 1,\dots,N \qquad (2.6)$$

$$D^{(S)}_{ij} = \int_{S_i} dx \int_{S_j} d\xi \frac{\partial G(x;\xi)}{\partial n_x} \qquad (2.7)$$

and equation (2.3),

$$-\frac{1}{2}\phi_i A_i + \sum_{j=1}^{N} D^{(G)}_{ij}\phi_j = \sum_{j=1}^{N} G_{ij} V_j, \quad i = 1,\dots,N \qquad (2.8)$$

$$D^{(G)}_{ij} = \int_{S_i} dx \int_{S_j} d\xi \frac{\partial G(\xi;x)}{\partial n_\xi} \qquad (2.9)$$

where A_i is the area of the i-th panel. The integration with respect to the x-variable introduced by the Galerkin averaging, allows the matrices $D^{(S)}$, $D^{(G)}$ and G to preserve the symmetry properties of the corresponding operators in the continuous case. In particular, the matrix G is symmetric, thus

$$G_{ij} = G_{ji} \qquad (2.10)$$

and the matrices $D^{(G)}$ and $D^{(S)}$ are the transpose of each other, or

$$D^{(G)}_{ij} = D^{(S)}_{ji} \qquad (2.11)$$

The proof of (2.10) follows from the symmetry of the wave-source potential with respect to its arguments, and of (2.11) by exchanging the role of the dummy variables under the integral signs in (2.7) and transposing the i and j indices. Analogous results do not hold in the collocation method, where the corresponding matrix elements are obtained by replacing the integration with respect to the x-variable in (2.4)-(2.9) by the selection of a collocation point.

Let $A = \mathrm{diag}(A_i)$, and define,

$$D = -\frac{1}{2}A + D^{(G)} \qquad (2.12)$$

The solution for the velocity potential obtained from equations (2.4)-(2.7) in terms of the matrices A, D and G, is given by

$$\vec{\phi}^{(S)} = A^{-1}G(D^T)^{-1}A\,\vec{v} \qquad (2.13)$$

and the solution of (2.8) by

$$\vec{\phi}^{(G)} = D^{-1}G\,\vec{v} \qquad (2.14)$$

For an arbitrary normal-velocity vector $\vec{v}$, a necessary condition for the identity of the two solutions (2.13) and (2.14) is the equality of the matrix $D^{-1}G$ with the matrix $A^{-1}G(D^T)^{-1}A$, or equivalently the symmetry of the matrix

$$W = A\,D^{-1}G \qquad (2.15)$$

A proof that W is symmetric did not prove possible. Numerical experiments for a model problem in two dimensions in an infinite fluid revealed that the matrix W is "almost symmetric", meaning that elements with symmetric locations relative to the principal diagonal agreed to 2-3 significant digits. This suggests the proximity of the values for the velocity potential obtained from each method.

The hydrodynamic forces can be obtained from the solutions (2.13) and (2.14) by multiplying the velocity potential by the panel area A_i and the vector u_i which represents the "direction" of the force we are interested to evaluate. This operation is equivalent to a pre-multiplication of the velocity-potential vector by the vector $(A\vec{u})^T$. The resulting hydrodynamic force obtained form the source-distribution method is given by

$$H^{(S)} = \vec{u}^T W^T \vec{v} \qquad (2.16)$$

and from the direct solution of the Green integral equation,

$$H^{(G)} = \vec{u}^T W\,\vec{v} \qquad (2.17)$$

Both $H^{(S)}$ and $H^{(G)}$ are complex scalars. Three properties follow from equations (2.16) and (2.17) :

1) For the diagonal hydrodynamic coefficients $H_{kk} = A_{kk} - i\,B_{kk}/\omega$, $k = 1,\ldots,6$, the vectors u_j and v_j are equal to the values on the j-th panel of the unit vector $\vec{n}_k$ which points out of the fluid domain. In this case the hydrodynamic coefficients predicted by the two methods are identical, since

$$\vec{v}^T W\,\vec{v} = \vec{v}^T W^T \vec{v} \qquad (2.18)$$

2) For the off-diagonal coefficients, it follows from (2.16) and (2.17) that

$$H^{(S)}_{K\ell} = H^{(G)}_{\ell K} \qquad (2.19)$$

3) For the diffraction exciting force, we define

$$v_j = -\left(\frac{\partial\phi_o}{\partial n}\right)_j \qquad (2.20)$$

where ϕ_o is the incident-wave velocity potential. If X_k is the diffraction exciting force in the k-th direction, it can be deduced from (2.16) and (2.17) that the force predicted by the source-distribution/Green method by directly solving the diffraction problem, is identical to the force obtained from the use of the Haskind relation with the velocity potential supplied by the solution of the Green/source-distribution integral equation.

Computations of the hydrodynamic coefficients of a spheroid and a vertical circular cylinder using both methods have been carried out by Breit, Newman and Sclavounos (1985). A radiation-diffraction computer program has been written for the hydrodynamic analysis of bodies of general shape. Their wetted surface is approximated by a collection of plane quadrilaterals and triangles, as illustrated in Figures 1 and 2 for a quarter of the spherroid and the vertical cylinder respectively.

For inter-panel distances small compared to their characteristic dimensions, the Rankine singularity (including when appropriate its image with respect to the free surface and the sea bottom) is subtracted from the wave-source potential and integrated analytically over the panels. For large distances between the panels the total wave-source potential is integrated by quadrature. The algorithms for the integration of the Rankine singularities on plane quadrilaterals and for the evaluation of the wave-source potential have been developed by Newman (1985a & b) and coded in the subroutines FINGREEN and RPAN respectively. The four-node Gauss-Legendre quadrature, adapted to a plane quadrilateral of general shape, has been used for the evaluation of influence coefficients in both the collocation and Galerkin methods. Suggestions on the order of the quadrature to be used in a production radiation-diffraction computer program are discussed at the end of the section.

The hydrodynamic-force predictions of the collocation and Galerkin methods did not differ substantially away from the irregular frequencies. Tabulated results of high accuracy are reported in Breit, Newman and Sclavounos. Figures 3 and 4 illustrate the behaviour of the two methods at the first heave irregular frequency of the spheroid and the cylinder respectively. The solid lines represent the predictions of an independent curvilinear-panel program for the spheroid, and a Fourier-transformed time-domain solution for the cylinder. The Galerkin predictions appear to be less sensitive to the irregular-frequency errors, especially for the spheroid coefficients.

The Galerkin technique requires an additional integration for the evaluation of each element in the D_{ij} and G_{ij} matrices, [eq. (2.4), (2.9)] relative to the collocation method. It is reasonable to assume that the accuracy in the integration over the i-th panel needs to be no higher than that over the j-th panel. Concerning the Rankine source and dipole, analytical expressions for the double integral over a pair of plane quadrilaterals are not known to the authors. When the analytical expressions are utilized for the evaluation of the Rankine source and dipole integrals over the j-th panel, a four-node Gauss-Legendre quadrature is suggested for the integration over the i-th panel. Since this result is frequency-independent it may be evaluated once and stored.

The integrals of the remaining slowly-varying but frequency-dependent parts, can be evaluated using a quadrature scheme of the same order for the j-th and i-th panels. In the collocation method, the use of a four-node Gauss quadrature causes an increase by a factor of four in the number of evaluations of the wave-source potential, versus the single-node centroid integration. This factor may be as high as sixteen in the Galerkin method. Optimality requires that errors due to quadrature and the approximation of the geometry by plane panels must be of comparable magnitude. This may be achieved by increasing the number of panels and utilizing a single-node quadrature. This decision depends on the efficiency in the evaluation of the wave-source potential, the solution of the linear system and the computing environment. If the single-node-quadrature route is selected, the collocation and Galerkin methods are comparably expensive over a large number of frequencies.

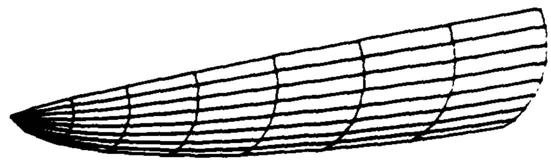

Figure 1 : Discretization of a quarter of the surface of a prolate sphepoid (B/L = 1/8) by 64 panels.

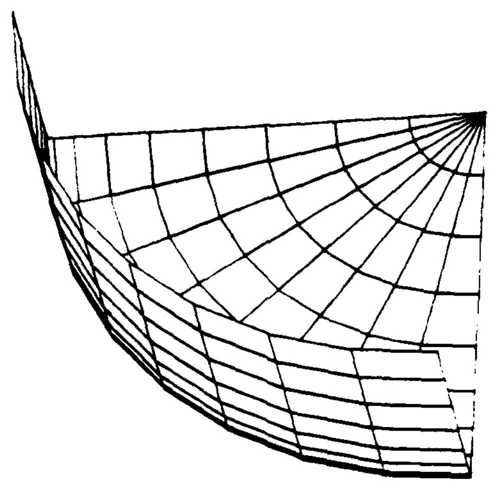

Figure 2 : Discretization of a quarter of the wetted surface of a truncated vertical cylinder (R/T = 2) by 128 panels.

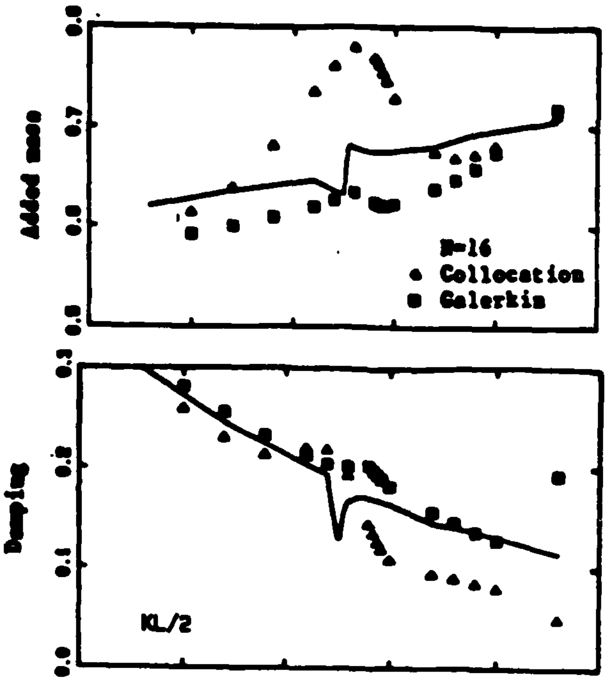

Figure 3 : Heave added-mass and damping coefficients of a prolate spheroid (B/L = 1/8) near the first irregular frequency, made non-dimensional by the displaced volume the water density and the frequency of oscillation. The solid line is obtained from an independent curvilinear-panel program and the tick marks are predictions form the plane-panel program. [From Breit, Newman and Sclavounos (1985)]

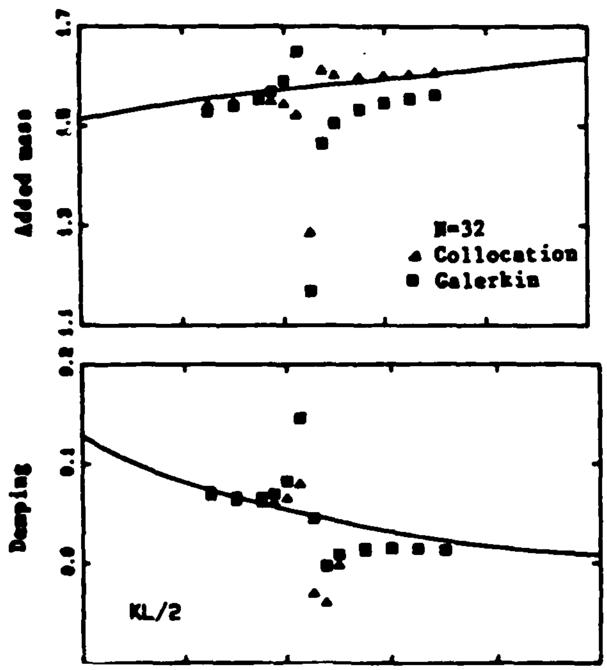

Figure 4 : Heave added-mass and damping coefficients of a truncated vertical cylinder near the first irregular frequency, made non-dimensional by the displaced volume the water density and the frequency of oscillation. The solid line is obtained from the Fourier transform of an independent time-domain program and the tick marks are predictions form the plane-panel program. [From Breit, Newman and Sclavounos (1985)]

3. Irregular frequencies

The Green integral equation (2.3) is known to possess non-vanishing homogeneous solutions at a discrete set of frequencies which correspond to the eigenfrequencies of the interior Dirichlet problem. Their detrimental effect in the numerical predictions of the added-mass and damping coefficients of surface-piercing bodies has been illustrated in Figures 3 and 4.

Although a discrete set in the continuous case, their presence in the discrete problem is manifested by substantial errors, often over a quite wide frequency band around their exact location. This is due to the bad "conditioning" of the integral equation (2.3) not only at, but also in the vicinity of the irregular frequencies. Bad conditioning is known to cause large errors in the solution when a small perturbation is imposed on the equation. In wave-body interactions sources of such perturbations are:

1) The approximation of the body boundary.

2) The approximation of the velocity potential.

3) Errors in the evaluation of the wave-source potential.

4) Quadrature errors in the evaluation of the influence coefficients.

5) The approximate way in which the equation is being satisfied.

6) Roundoff errors in the solution of the linear system.

A measure of the ratio of the output versus the input errors in the solution of integral equations is often supplied by the "condition number". Explicit definitions of it are known for matrix equations. Thus the discrete form of an integral equation may be used to obtain an estimate of it.

Numerical experiments indicate that the errors and frequency bandwidth of the irregular frequencies decrease with increasing numbers of panels. The associated computational cost, however, prevents this to be considered a viable treatment in practice. A short survey of successful methods for the removal of the irregular-frequency effects has been given in the Introduction.

The method of Burton and Miller (1971), developed for the solution of an acoustic scattering integral equation, is here adapted to the wave-body interaction problem. The Green equation (2.3) is valid for a point x on the body boundary. If x lies in the fluid domain, the factor 1/2 which multiplies the first term needs to be replaced by unity. Taking the derivative of both sides in the direction of the unit vector $\hat{n}$ which points out of the fluid domain, and letting the point x approach the body boundary, we obtain

$$-\frac{\partial\phi(x)}{\partial n_x} + \frac{\partial}{\partial n_x}\int_S \phi(\xi)\,\frac{\partial G(\xi;x)}{\partial n_\xi}\,d\xi =$$

$$-\frac{1}{2}\frac{\partial\phi}{\partial n_x} + \int_S \frac{\partial\phi}{\partial n_\xi}\,\frac{\partial G(x;\xi)}{\partial n_x}\,d\xi \qquad (3.1)$$

For a prescribed normal velocity, (3.1) is an integral equation of the first kind for the velocity potential on the body boundary. Its irregular frequencies correspond to the eigenfrequencies of the interior Neumann problem. Burton and Miller (1971) show that the linear combination (2.3) + i α(3.1), or

$$-\frac{1}{2}\phi(x) + \int_S \phi(\xi)\frac{\partial}{\partial n_\xi}\left(1 + i\alpha\frac{\partial}{\partial n_x}\right)G(\xi;x)\,d\xi =$$

$$\frac{i\alpha}{2}\frac{\partial\phi}{\partial n_x} + \int_S \frac{\partial\phi}{\partial n_\xi}\left(1 + i\alpha\frac{\partial}{\partial n_x}\right)G(x;\xi)\,d\xi \qquad (3.2)$$

has no real irregular frequencies for real and positive values of the parameter α . Numerical experimentation suggests that for values of the parameter α ranging from 0.2 to 0.3, the performance of (3.2) is optimal. A discussion of the properties of equations (2.3), (3.1) and (3.2) for acoustic-wave problems, is given by Colton and Kress (1983).

Approximating the body boundary by a collection of panels S_j, and satisfying (3.2) at collocation points located at the panel centroids, we obtain the discrete set of equations

$$-\frac{1}{2}\phi_i + \sum_{j=1}^{N}\phi_j\int_{S_j}\frac{\partial}{\partial n_\xi}\left(1 + i\alpha\frac{\partial}{\partial n_{x_i}}\right)G(\xi;x_i)\,d\xi$$

$$= \frac{i\alpha}{2}V_i + \sum_{j=1}^{N}V_j\int_{S_j}\left(1 + i\alpha\frac{\partial}{\partial n_{x_i}}\right)G(x_i;\xi)\,d\xi\ ,$$

$$i = 1,\dots,N \qquad (3.3)$$

The integral of the double normal derivative of the singular part of the wave-source potential needs careful interpretation For i=j, it is equal to the normal velocity on the panel due to a distribution of dipoles of constant strength on its surface. This value is known to exist at points not lying on its edges.

The numerical conditioning of equation (3.1) is worse than that of the Green equation (2.3). This is generally known to be true for integral equations of the first kind. Hence, the perturbations 1-6 cause errors in the solution of (3.1) large relative to those in the solution of (2.3).

The effect of equation (3.1) on the solution of (3.2) is controlled by the magnitude of the positive quantity α . If α =0, (3.2) reduces to the Green equation. For finite values of α , (3.2) is expected to be free of irregular frequency effects. This turns out not to be the case in practice for very small values of α which damp excessively the effect of equation (3.1). In this case the error in the hydrodynamic hydrodynamic coefficients near the irregular frequencies of (2.3), although reduced, is still substantial. For large values of α , the effects of equation (3.1) are dominant. The predicted coefficients are now erroneous near its own irregular frequencies. Smaller, but still noticeable, errors are also present for all frequencies due to its poor conditioning. The magnitude of both can be obviously controlled by the selection of a sufficiently small value of α which strikes a balance between the errors coming from the irregular frequencies of the Green equation (2.3) and those coming from the ill-conditioning of equation (3.1).

Computations of the heave and sway coefficients of a circle and a rectangle (B/T=2), indicate that the value of α =0.2 produces satisfactory results over a wide range of frequencies. A value of less than one is not surprising. If the error in the solution of (3.2) resulting from the presence of equations (2.3) is to be comparable to the error due to equation (3.1), the value of α must be comparable to the ratio of their condition numbers which is a quantity with magnitude less than one.

A computer program has been written for the hydrodynamic analysis of sections of general shape in deep water [Sclavounos (1985)]. The computational effort involved in the set-up and solution of the discrete equations (3.3), is for all practical purposes comparable to that required when α =0. This is due to the existence of recurrence relations which express higher derivatives of the wave-source potential in terms of derivatives of lower order. The size of the linear system is unaffected by the addition of (3.1) to the Green equation.

Figure 5 presents the heave hydrodynamic coefficients of the circle and the rectangle near the first irregular frequency of equation (2.3). Predictions for α =0, 0.02 and 0.2 are compared to those obtained from the hybrid-integral solution of Nestegard and Sclavounos (1984) which is known to be free of irregular-frequency effects. The corresponding results for sway are presented in Figure 6 with analogous conclusions.

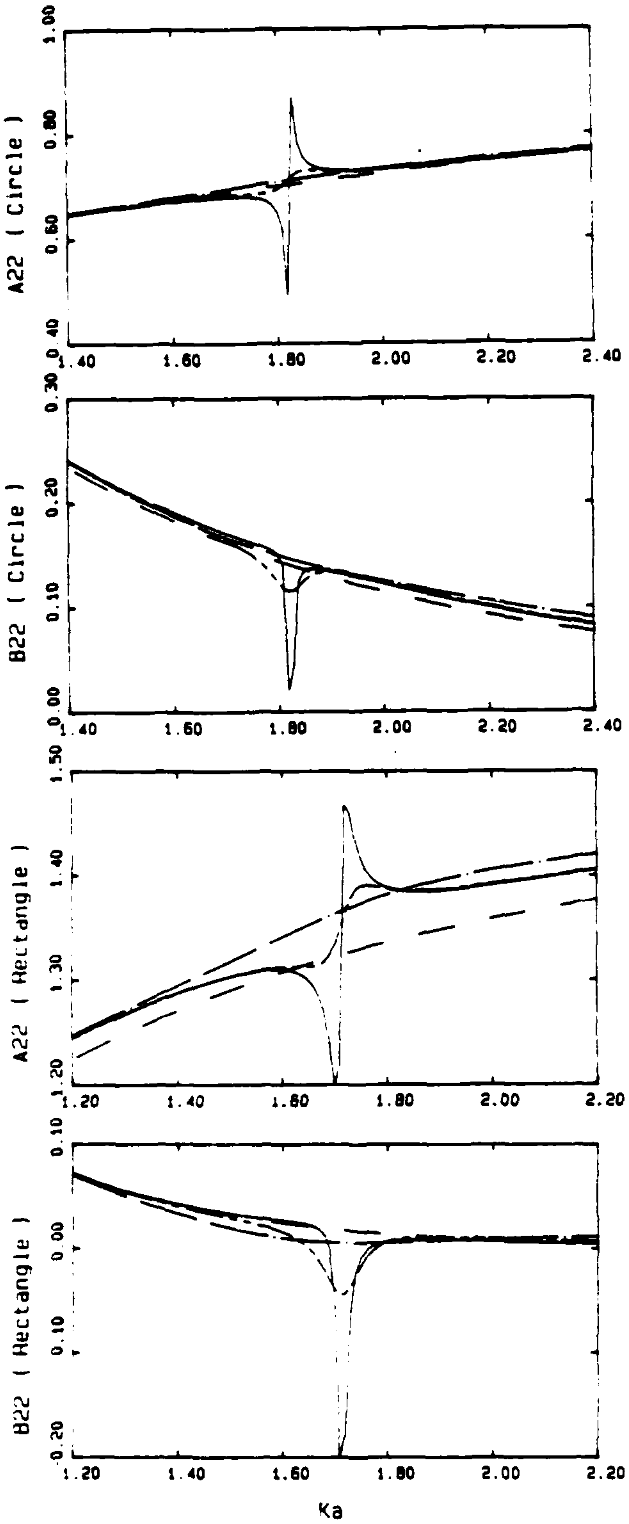

Figure 5

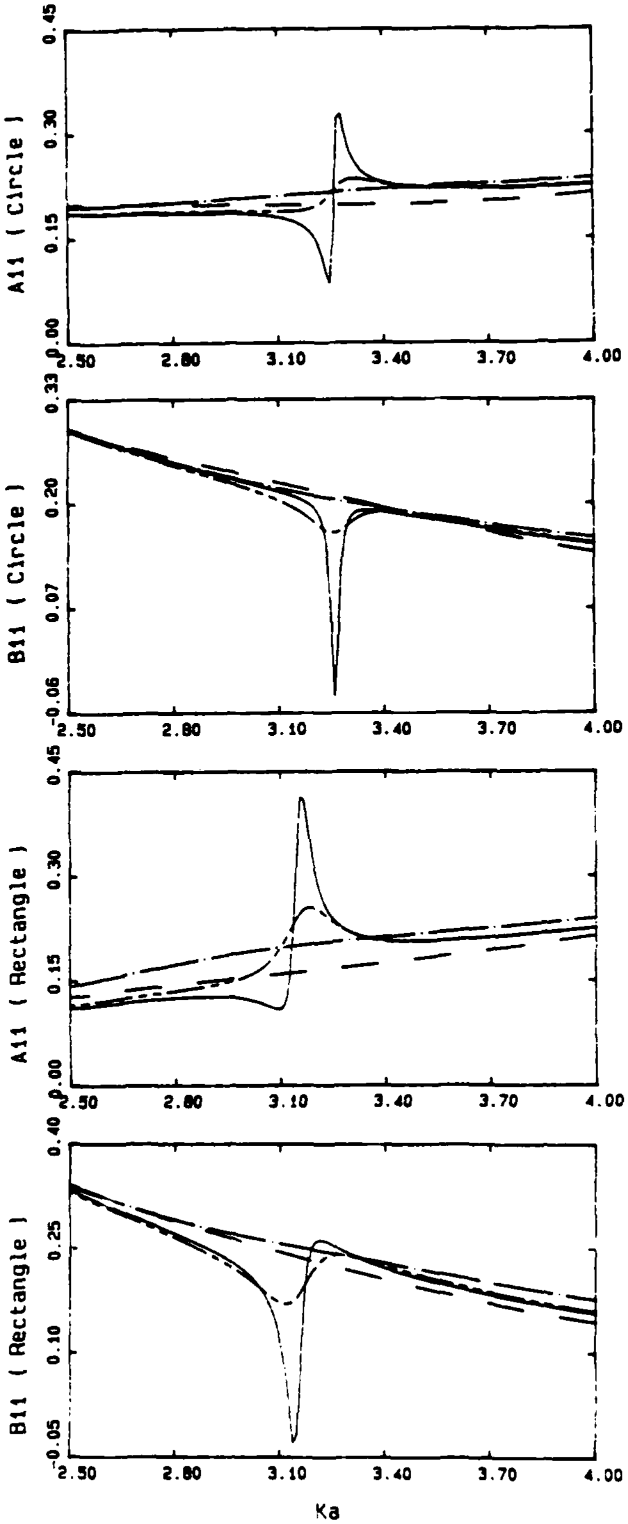

Figure 6

Heave and sway added-mass and damping coefficients near the first irregular frequency of the Green equation (2.3), obtained from the solution of equation (3.3) : (——) α =0 ; (—··—) α =0.02 ; (—·—) α =0.2; (— —) hybrid-integral solution of Nestegard and Sclavounos (1984).

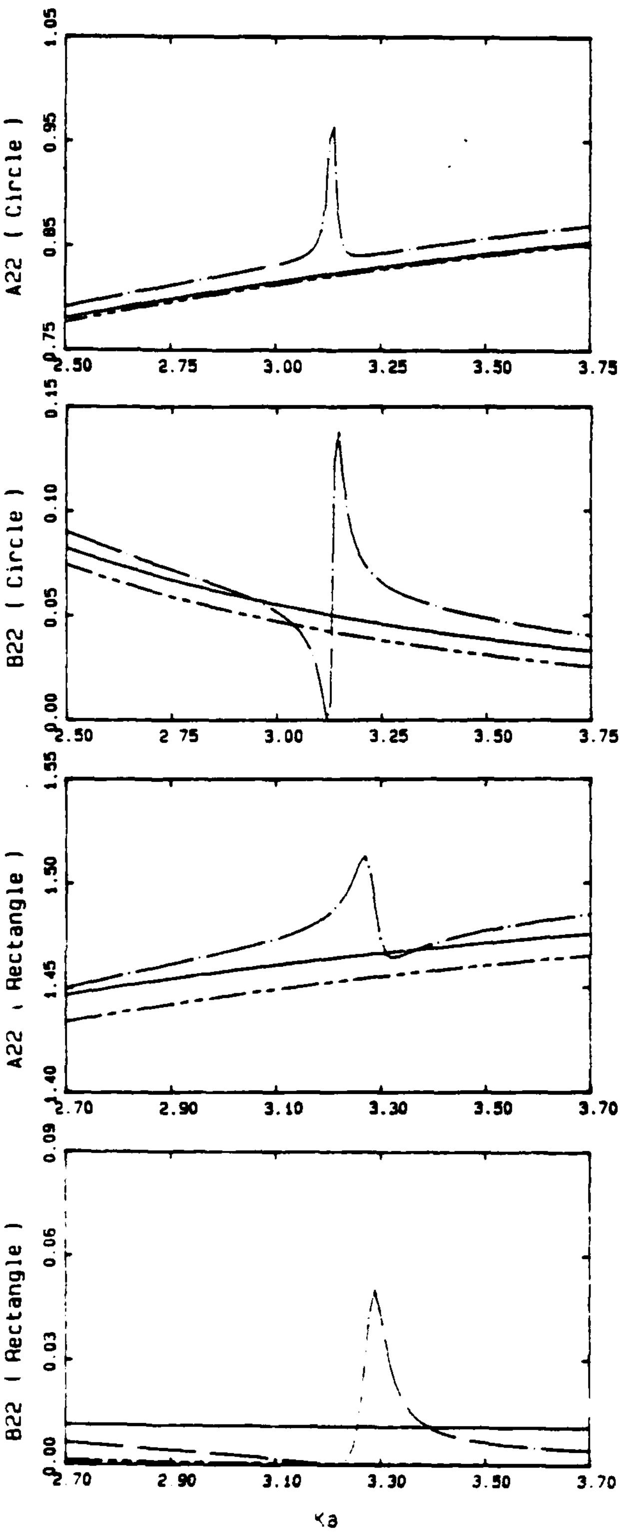

Figure 7

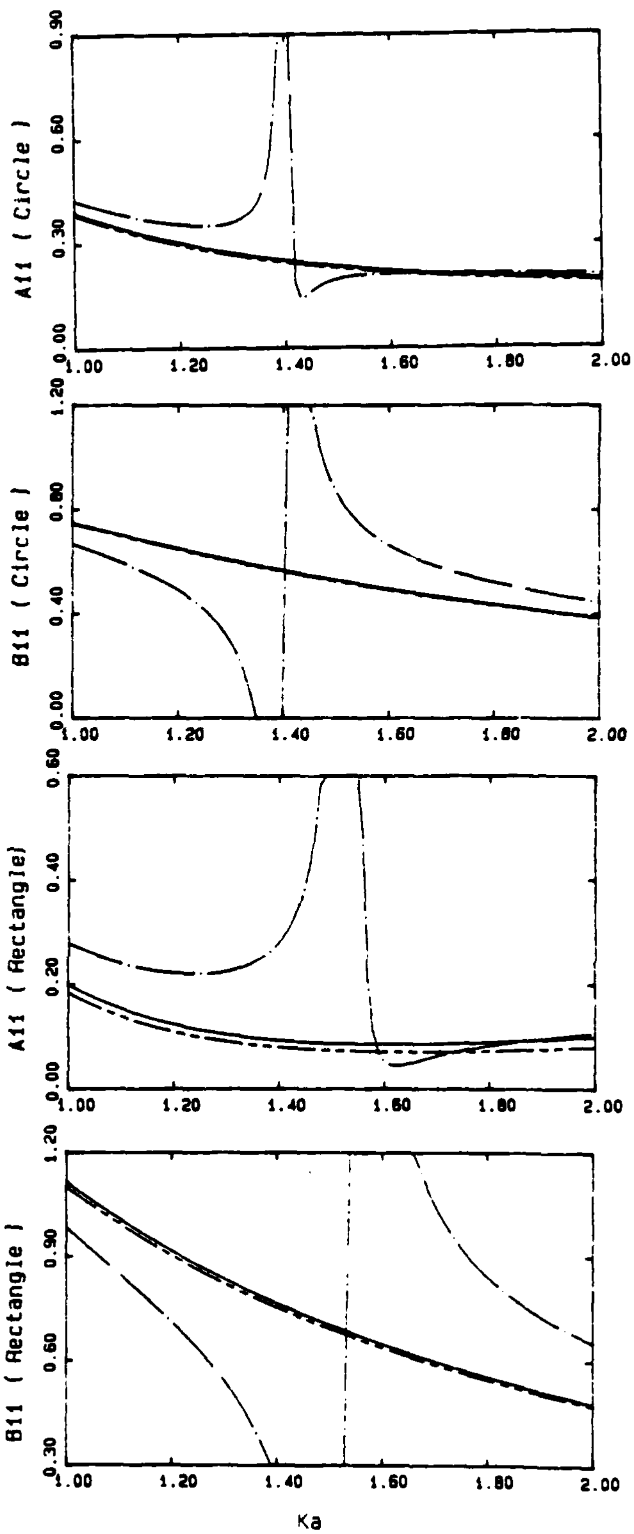

Figure 8

Heave and sway added-mass and damping coefficients near the first irregular frequency of equation (3.1), obtained from the solution of equation (3.3) : (———) α =0 ; (— - - —) α =0.2 ; (—·—) equation (3.1).

Figures 7 and 8 illustrate the performance of equation (3.3) near the first irregular frequency of equation (3.1). Results are presented for $\alpha = 0$, 0.2 and from the solution of equation (3.1) alone. The predictions for $\alpha=0$ are accurate over that frequency range, and are in good agreement with the results for $\alpha = 0.2$. The larger discrepancies occur in the heave damping coefficients, but these are probably due to its small values. The predictions from equation (3.1) display the excpected error near its first irregular frequency. Evident is also a non-negligible error over a wider frequency range, mainly in the sway coefficients. This is larger than the corresponding error associated with equation (2.3), and is due to the larger condition number of equation (3.1). The effect of (3.1) in the composite equation (3.3) is here reduced by the selection of a value for α equal to 0.2.

In all cases tested, half of the boundary of the sections analysed has been approximated by 10 straight segments. Away from the irregular frequencies, the agreement between the coefficients obtained from equation (3.3) for $\alpha = 0.2$ and the numerical scheme of Nestegard and Sclavounos is very good. Work is currently in progress for the extention of the present method to three dimensions.

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